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PARALLEL ALGORITHM FOR IDENTIFICATION OF PARAMETERS OF THE MODEL OF SUSPENSION FILTRATION IN A POROUS MEDIUM

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The paper explores a mathematical model for suspension filtration in a porous medium, incorporating a mass balance equation for suspended particles and kinetic equations for both irreversible and reversible particle deposition. An inverse problem was formulated and solved numerically to determine four parameters of the model at once. Four parameters to find: the diffusion coefficient in the mass balance equation, deposition rate coefficients in the kinetic equations of both active and passive zones and reversible deposition re-entrainment coefficient. A first-order identification method was used for this purpose. The results show that when the initial approximations are close to the exact values of given parameters, the parameters are recovered with a small number of iterations. When the initial approximations deviate slightly from the given values, the number of iterations required to recover the parameters increases, but the coefficients are recovered with a sufficiently small error. It was found that when the initial approximations of the parameters are sufficiently far from the exact values of given parameters, the first-order identification method does not give good results, and the iterative process becomes divergent. In this case, a modified identification method using regularization was used to recover the parameters, and the parameters were recovered with sufficient accuracy. Taking into account that a large amount of calculations are performed during the inverse problem, a parallel algorithm was proposed for processing this problem. It was found that the program based on the parallelized algorithm works significantly faster than the original program.

Keywords: filtration, finite differences, inverse problem, mathematical model, parallel algorithm, porous medium, regularization.

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1 Introduction

Waterflooding is a predominant method for global oil production, involving the injection of water into certain wells and the extraction of oil from others [1]. However, the injection of untreated water, containing various organic and mineral additives, has been observed to diminish the water-bearing capacity of the reservoir [2]. Introducing low-quality water into wells during waterflooding diminishes permeability due to the entrapment of suspended particles, forming deposits as the liquid traverses the porous medium [3].

In the realm of mathematical models for filtration processes, functions describing the properties of the porous medium or the fluid in which the flow occurs are integral [2].

Various authors have developed methods for indirectly recovering crucial parameters, such as pressure or flow velocity, from laboratory measurements in flow experiments [4–7]. These approaches give rise to inverse problems in mathematical physics within the realm of parameter identification theory, presenting challenges as ill-conditioned linear and nonlinear optimization problems. To address these challenges, regularization methods prove valuable, ensuring the stability of approximate solutions [6, 7].

While the direct problems related to solute transport in porous media have received considerable attention, the inverse problems, even for the simplest models, remain inadequately explored [8]. This article delves into the inverse problem of identifying parameters in models of inhomogeneous fluid filtration within porous media.

Flow of water containing suspended particles through porous media, cause the deposition of particles, leading to a reduction in medium permeability. This phenomenon known as deep bed filtration. Deep bed filtration play crucial roles in various petroleum-related applications, such as controlling sand production, managing fines migration, disposing of produced water in aquifers, and implementing deep bed filtration in gravel packs. Particle suspension filtration is not limited to petroleum applications; it occurs in industrial water filtering, the propagation of contaminants (including viruses and bacteria) through aquifers, and various environmental processes. Deep bed filtration of particle suspensions in porous media takes place during activities like water injection into oil reservoirs, drilling fluid invasion of reservoir production zones, fines migration in oil fields, industrial filtering, and the transport of bacteria, viruses, or contaminants in groundwater.

2 Problem formulation

Let we consider a layer with length L and initial porosity m_0 filled with a homogeneous liquid. From $t > 0$ to $t \leqslant t_1$ at the point $x = 0$ the inhomogeneous liquid containing suspended solid particles with concentration c_0 starts injecting to the layer with the constant filtration velocity U.

Suspension filtration occurs in porous media with active and passive zones, which means deposition formation, have reversible and irreversible forms, respectively. In this case, mathematical model of the process consists of mass balance equation and kinetic equations for both zones [9, 10]

$$\frac{\partial c}{\partial t} + U\frac{\partial c}{\partial x} + \frac{d_m}{m}\frac{\partial \rho_a}{\partial t} + \frac{d_m}{m}\frac{\partial \rho_p}{\partial t} = D_L\frac{\partial^2 c}{\partial x^2}, \tag{1}$$

$$\frac{d_m}{m}\frac{\partial \rho_a}{\partial t} = \beta_a c - \beta_d\frac{d_m}{m}\rho_a, \tag{2}$$

$$\frac{d_m}{m}\frac{\partial \rho_p}{\partial t} = \beta_p c, \tag{3}$$

where c is concentration of suspended particles in the fluid, d_m is the bulk density, m is the porosity, ρ_a and ρ_p are deposited particle concentration in active and passive zones, respectively, D_L is diffusion coefficient, β_a is reversible deposition rate coefficient, β_d is reversible deposition re-entrainment coefficient, β_p is irreversible deposition rate coefficient, x is the coordinate.

Initial and boundary conditions are following

$$c(x,0) = 0, \quad \rho_a(x,0) = \rho_p(x,0) = 0, \tag{4}$$

$$\begin{aligned} &c(0,t) = \begin{cases} c_0, & 0 \leqslant t \leqslant t_1, \\ 0, & t > t_1, \end{cases} \\ &\left.\tfrac{\partial c}{\partial t}\right|_{x=L} = 0. \end{aligned} \tag{5}$$

The problem (1)-(5) corresponds to the direct statement. With known all coefficients in (1)-(3) and as well as c_0 in (5), the solutions c, ρ_a, ρ_p. When solving the coefficient inverse problem, some coefficients in (1)-(3) are unknown and must be determined. In this paper we consider an inverse problem, which consists of determining the coefficients D_L, β_a, β_d, β_p in equations (1)-(3). This requires additional information about the solution of the direct problem. To do this, we solve the direct problem with known values of the parameters. As additional information we use values of the effluent suspended particles concentration in n different time points, which we denote as $z(t_j)$, where

$$c(L, t_j) = z(t_j), \;\; j = \overline{1, n}, \tag{6}$$

where n is number of selected points of time.

3 Solution of the problem

3.1 Solution of the direct problem

To solve the direct problem (1)-(5), we use finite difference method. In the area $D = \{0 \leqslant x \leqslant L, \;\; 0 \leqslant t \leqslant T\}$, we consider the net

$$\omega_{h\tau} = \{(x_k,\, t_j)\,,\; x_k = kh,\, k = 0, 1, ..., K,\;\; h = L/K,\, t_j = j\tau,\;\; j = 0, 1, ..., J,\, \tau = T/J\}\,.$$

Instead of functions $c\,(t, x)$, $\rho_a\,(t, x)$, $\rho_p\,(t,\, x)$, we consider grid functions, whose values at the nodes $(x_k,\, t_j)$ are denoted by $(c)_k^j$, $(\rho_a)_k^j$, $(\rho_p)_k^j$, respectively.

Difference scheme for the equation (2) is

$$\frac{d_m}{m}\frac{\rho_{a,i}^{j+1} - \rho_{a,i}^{j}}{\tau} = \beta_a c_i^j - \frac{d_m}{m}\beta_d \rho_{a,i}^j,$$

after simple transformations, the scheme takes the form

$$\rho_{a,i}^{j+1} = \rho_{a,i}^{j} + \tau\left(\frac{m}{d_m}\beta_p c_i^j - \beta_d \rho_{a,i}^j\right). \tag{7}$$

For equation (3) we have

$$\frac{d_m}{m}\frac{\rho_{p,i}^{j+1} - \rho_{p,i}^{j}}{\tau} = \beta_p c_i^j,$$

after simple transformations, the scheme takes the form

$$\rho_{p,i}^{j+1} = \rho_{p,i}^{j} + \tau\frac{m}{d_m}\beta_p c_i^j. \tag{8}$$

The balance equations(1) is approximated by the equations

$$\frac{c_i^{j+1} - c_i^j}{\tau} + U\frac{c_i^{j+1} - c_{i-1}^{j+1}}{h} + \frac{d_m}{m}\frac{\rho_{a,i}^{j+1} - \rho_{a,i}^{j}}{\tau} + \frac{d_m}{m}\frac{\rho_{p,i}^{j+1} - \rho_{p,i}^{j}}{\tau} = D\frac{c_{i-1}^{j+1} - 2c_i^{j+1} + c_{i+1}^{j+1}}{h^2}. \tag{9}$$

We also approximate initial and boundary conditions (4), (5)

$$c_i^0 = 0, \rho_{a,i}^0 = \rho_{p,i}^0 = 0, i = \overline{0, I}, \tag{10}$$

$$c_0^j = \begin{cases} c_0, & 0 \leqslant t^j \leqslant t_1, \\ 0, & t^j > t_1, \end{cases}$$

$$c_N^j = c_{N-1}^j,\ j = \overline{0, J}. \tag{11}$$

We transform the scheme (9) to obtain the following system of linear equations

$$Ac_{i-1}^{j+1} - Cc_i^{j+1} + Bc_{i+1}^{j+1} = -F_i^j, \tag{12}$$

where

$$A = \frac{D\tau}{h^2} + \frac{U\tau}{h}, \quad C = \frac{2D\tau}{h^2} + \frac{U\tau}{h} + 1,$$

$$B = \frac{D\tau}{h^2},\ F_i^j = c_i^j - \frac{d_m}{m}\left(\rho_{a,i}^{j+1} - \rho_{a,i}^j\right) - \frac{d_m}{m}\left(\rho_{p,i}^{j+1} - \rho_{p,i}^j\right).$$

We solve (12) by the tridiagonal matrix algorithm (Thomas' algorithm)

$$c_i^{j+1} = \alpha_{i+1}c_{i+1}^{j+1} + \beta_{i+1}, \tag{13}$$

where

$$\alpha_{i+1} = \frac{B}{C - A\alpha_i}, \quad \beta_{i+1} = \frac{A\beta_i + F_i^j}{C - A\alpha_i}.$$

3.2 Solution of the inverse problem

In this section we consider an inverse problem, which consists of determining the coefficients D_L, β_a, β_d, β_p in equations (1)-(3), with given additional information (6).

We determine these coefficients by minimizing the following functional

$$\Phi\left(D_L, \beta a, \beta_d, \beta_p\right) = \int_0^T \left[c\left(L, t\right) - z\left(t\right)\right]^2 dt. \tag{14}$$

The stationarity condition of the functional (14) is as follows [8, 11-14]

$$\frac{d\Phi\left(\gamma\right)}{d\gamma} = 2\int_0^T \left[c\left(L, t\right) - z_l\left(t\right)\right] w\left(L, t\right) dt = 0, \tag{15}$$

where w is vector column, γ is row vector and

$$w = \frac{dc}{d\gamma} = \left(w_{11}, w_{12}, w_{13}, w_{14}\right)^T,\ \gamma = \left(D_L, \beta_a, \beta_d, \beta_p\right). \tag{16}$$

We expand the function $c\left(x, t\right)$ in a neighborhood of γ^s (here and other places of the text s is number of iteration) up to second-order terms [11]

$$c^{s+1}\left(x, t\right) \approx c^s\left(x, t\right) + \left(\gamma^{s+1} - \gamma^s\right) \cdot w^s\left(x, t\right). \tag{17}$$

Substituting (17) to (14), we get following equation with respect to D_L^{s+1}, β_a^{s+1}, β_d^{s+1}, β_p^{s+1}

$$\left\{\begin{array}{l} a_{11}D_L^{s+1} + a_{12}\beta_a^{s+1} + a_{13}\beta_d^{s+1} + a_{14}\beta_p^{s+1} = b_1, \\ a_{21}D_L^{s+1} + a_{22}\beta_a^{s+1} + a_{23}\beta_d^{s+1} + a_{24}\beta_p^{s+1} = b_2, \\ a_{31}D_L^{s+1} + a_{32}\beta_a^{s+1} + a_{33}\beta_d^{s+1} + a_{34}\beta_p^{s+1} = b_3, \\ a_{41}D_L^{s+1} + a_{42}\beta_a^{s+1} + a_{43}\beta_d^{s+1} + a_{44}\beta_p^{s+1} = b_4, \end{array}\right. \tag{18}$$

where

$$a_{11} = \int_0^T \left(w_{11}^s(L, t)\right)^2 dt,\ a_{22} = \int_0^T \left(w_{12}^s(L, t)\right)^2 dt,$$

$$a_{33} = \int_0^T (w_{13}^s(L,t))^2\, dt,\ a_{44} = \int_0^T (w_{14}^s(L,t))^2\, dt,$$

$$a_{12} = a_{21} = \int_0^T w_{11}^s(L,t) \cdot w_{12}^s(L,t)\, dt,\ a_{13} = a_{31} = \int_0^T w_{11}^s(L,t) \cdot w_{13}^s(L,t)\, dt,$$

$$a_{23} = a_{32} = \int_0^T w_{12}^s(L,t) \cdot w_{13}^s(L,t)\, dt,\ a_{14} = a_{41} = \int_0^T w_{11}^s(L,t) \cdot w_{14}^s(L,t)\, dt,$$

$$a_{24} = a_{42} = \int_0^T w_{12}^s(L,t) \cdot w_{14}^s(L,t)\, dt,\ a_{34} = a_{43} = \int_0^T w_{13}^s(L,t) \cdot w_{14}^s(L,t)\, dt, \tag{19}$$

$$b_1 = \int_0^T \left[w_{11}^s(L,t)\, D_L^s + w_{12}^s(L,t)\, \beta_a^s + w_{13}^s(L,t)\, \beta_d^s + w_{14}^s(L,t)\, \beta_p^s - c^s(L,t) + z(t)\right] w_{11}^s(L,t)\, dt,$$

$$b_2 = \int_0^T \left[w_{11}^s(L,t)\, D_L^s + w_{12}^s(L,t)\, \beta_a^s + w_{13}^s(L,t)\, \beta_d^s + w_{14}^s(L,t)\, \beta_p^s - c^s(L,t) + z(t)\right] w_{12}^s(L,t)\, dt,$$

$$b_3 = \int_0^T \left[w_{11}^s(L,t)\, D_L^s + w_{12}^s(L,t)\, \beta_a^s + w_{13}^s(L,t)\, \beta_d^s + w_{14}^s(L,t)\, \beta_p^s - c^s(L,t) + z(t)\right] w_{13}^s(L,t)\, dt,$$

$$b_4 = \int_0^T \left[w_{11}^s(L,t)\, D_L^s + w_{12}^s(L,t)\, \beta_a^s + w_{13}^s(L,t)\, \beta_d^s + w_{14}^s(L,t)\, \beta_p^s - c^s(L,t) + z(t)\right] w_{14}^s(L,t)\, dt.$$

Further approximations of D_L^{s+1}, β_a^{s+1}, β_d^{s+1}, β_p^{s+1} can be found by solving the system, we apply Cramer's rule to it

$$D_L^{s+1} = \frac{\Delta_{D_L}}{\Delta},\quad \beta_a^{s+1} = \frac{\Delta_{\beta_a}}{\Delta},\quad \beta_d^{s+1} = \frac{\Delta_{\beta_d}}{\Delta},\quad \beta_p^{s+1} = \frac{\Delta_{\beta_p}}{\Delta}, \tag{20}$$

where

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix},\quad \Delta_{D_L} = \begin{vmatrix} b_1 & a_{12} & a_{13} & a_{14} \\ b_2 & a_{22} & a_{23} & a_{24} \\ b_3 & a_{32} & a_{33} & a_{34} \\ b_4 & a_{42} & a_{43} & a_{44} \end{vmatrix},\quad \Delta_{\beta_a} = \begin{vmatrix} a_{11} & b_1 & a_{13} & a_{14} \\ a_{21} & b_2 & a_{23} & a_{24} \\ a_{31} & b_3 & a_{33} & a_{34} \\ a_{41} & b_4 & a_{43} & a_{44} \end{vmatrix},$$

$$\Delta_{\beta_d} = \begin{vmatrix} a_{11} & a_{12} & b_1 & a_{14} \\ a_{21} & a_{22} & b_2 & a_{24} \\ a_{31} & a_{32} & b_3 & a_{34} \\ a_{41} & a_{42} & b_4 & a_{44} \end{vmatrix},\quad \Delta_{\beta_p} = \begin{vmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \\ a_{41} & a_{42} & a_{43} & b_4 \end{vmatrix}.$$

We differentiate (1)-(5) with respect to D_L

$$\frac{\partial w_{11}}{\partial t} + U\frac{\partial w_{11}}{\partial x} + \frac{d_m}{m}\frac{\partial w_{21}}{\partial t} + \frac{d_m}{m}\frac{\partial w_{31}}{\partial t} = D_L\frac{\partial^2 w_{11}}{\partial x^2} + \frac{\partial^2 c}{\partial x^2}, \tag{21}$$

$$\frac{d_m}{m}\frac{\partial w_{21}}{\partial t} = \beta_a w_{11} - \beta_d \frac{d_m}{m} w_{21}, \tag{22}$$

$$\frac{d_m}{m}\frac{\partial w_{31}}{\partial t} = \beta_p w_{11}, \tag{23}$$

$$w_{11}(x,0) = w_{21}(x,0) = w_{31}(x,0) = 0, \tag{24}$$

$$w_{11}(0,t) = 0, \qquad \left.\frac{\partial w_{11}}{\partial x}\right|_{x=L} = 0, \tag{25}$$

where $w_{11} = \frac{\partial c}{\partial D_L}$, $w_{21} = \frac{\partial \rho_a}{\partial D_L}$, $w_{31} = \frac{\partial \rho_p}{\partial D_L}$ are sensitivity functions with respect to D_L.

We differentiate (1)-(5) with respect to β_a and get following

$$\frac{\partial w_{12}}{\partial t} + U\frac{\partial w_{12}}{\partial x} + \frac{d_m}{m}\frac{\partial w_{22}}{\partial t} + \frac{d_m}{m}\frac{\partial w_{32}}{\partial t} = D_L\frac{\partial^2 w_{12}}{\partial x^2}, \tag{26}$$

$$\frac{d_m}{m}\frac{\partial w_{22}}{\partial t} = \beta_a w_{12} - \beta_d\frac{d_m}{m}w_{22} + c, \tag{27}$$

$$\frac{d_m}{m}\frac{\partial w_{32}}{\partial t} = \beta_p w_{12}, \tag{28}$$

$$w_{12}(x,0) = w_{22}(x,0) = w_{32}(x,0) = 0, \tag{29}$$

$$w_{12}(0,t) = 0, \qquad \left.\frac{\partial w_{12}}{\partial x}\right|_{x=L} = 0, \tag{30}$$

where $w_{12} = \frac{\partial c}{\partial \beta_a}$, $w_{22} = \frac{\partial \rho_a}{\partial \beta_a}$, $w_{32} = \frac{\partial \rho_p}{\partial \beta_a}$ are sensitivity functions with respect to β_a.

Then we differentiate (1)-(5) with respect to β_d and obtain following

$$\frac{\partial w_{13}}{\partial t} + U\frac{\partial w_{13}}{\partial x} + \frac{d_m}{m}\frac{\partial w_{23}}{\partial t} + \frac{d_m}{m}\frac{\partial w_{33}}{\partial t} = D_L\frac{\partial^2 w_{13}}{\partial x^2}, \tag{31}$$

$$\frac{d_m}{m}\frac{\partial w_{23}}{\partial t} = \beta_a w_{13} - \beta_d\frac{d_m}{m}w_{23} - \frac{d_m}{m}\rho_a, \tag{32}$$

$$\frac{d_m}{m}\frac{\partial w_{33}}{\partial t} = \beta_p w_{13}, \tag{33}$$

$$w_{13}(x,0) = w_{23}(x,0) = w_{33}(x,0) = 0, \tag{34}$$

$$w_{13}(0,t) = 0, \qquad \left.\frac{\partial w_{13}}{\partial x}\right|_{x=L} = 0. \tag{35}$$

where $w_{13} = \frac{\partial c}{\partial \beta_d}$, $w_{23} = \frac{\partial \rho_a}{\partial \beta_d}$, $w_{33} = \frac{\partial \rho_p}{\partial \beta_d}$ are sensitivity functions with respect to β_d.

Finally we differentiate (1)-(5) with respect to β_p

$$\frac{\partial w_{14}}{\partial t} + U\frac{\partial w_{14}}{\partial x} + \frac{d_m}{m}\frac{\partial w_{24}}{\partial t} + \frac{d_m}{m}\frac{\partial w_{34}}{\partial t} = D_L\frac{\partial^2 w_{14}}{\partial x^2}, \tag{36}$$

$$\frac{d_m}{m}\frac{\partial w_{24}}{\partial t} = \beta_a w_{14} - \beta_d\frac{d_m}{m}w_{24}, \tag{37}$$

$$\frac{d_m}{m}\frac{\partial w_{34}}{\partial t} = \beta_p w_{14} + c, \tag{38}$$

$$w_{14}(x,0) = w_{24}(x,0) = w_{34}(x,0) = 0, \tag{39}$$

$$w_{14}(0,t) = 0, \qquad \left.\frac{\partial w_{14}}{\partial x}\right|_{x=L} = 0. \tag{40}$$

The algorithm for determining coefficients D_L, β_a, β_d, β_p is as following:

1. We choose initial values of $D_L = D_L^0$, $\beta_a = \beta_a^0$, $\beta_d = \beta_d^0$ va $\beta_p = \beta_p^0$ (initially $s = 0$).

2. We solve problems (1)-(5), (21)-(25), (26)-(30), (31)-(35) and (36)-(40) from $t = 0$ to $t = T$ and determine functions $c^s(x,t)$, $\rho_a^s(x,t)$, $\rho_p^s(x,t)$, $w_{11}^s(x,t)$, $w_{21}^s(x,t)$, $w_{31}^s(x,t)$, $w_{12}^s(x,t)$, $w_{22}^s(x,t)$, $w_{32}^s(x,t)$, $w_{13}^s(x,t)$, $w_{23}^s(x,t)$, $w_{33}^s(x,t)$, $w_{14}^s(x,t)$, $w_{24}^s(x,t)$, $w_{34}^s(x,t)$.

3. We solve system of equations (18)-(20) and determine D_L^{s+1}, β_a^{s+1}, β_d^{s+1}, β_p^{s+1}.

4.$s = s+1, D_L = D_L^{s+1}$, $\beta_a = \beta_a^{s+1}$, $\beta_d = \beta_d^{s+1}$, $\beta_p = \beta_p^{s+1}$.

5. Repeat steps 2), 3), 4) until following conditions are satisfied

$$\left|\frac{\Phi^{s+1} - \Phi^s}{\Phi^s}\right| \leqslant \varepsilon; \left|\frac{D_L^{s+1} - D_L^s}{D_L^s}\right| \leqslant \varepsilon_1; \left|\frac{\beta_a^{s+1} - \beta_a^s}{\beta_a^s}\right| \leqslant \varepsilon_2; \left|\frac{\beta_d^{s+1} - \beta_d^s}{\beta_d^s}\right| \leqslant \varepsilon_3; \left|\frac{\beta_p^{s+1} - \beta_p^s}{\beta_p^s}\right| \leqslant \varepsilon_3.$$

where ε, ε_1, ε_2, ε_3 are fairly small values representing the accuracy of the solution.

3.3 Parallel algorithm for solving the inverse problem

In this paragraph, we consider the possibilities of using parallel computing algorithms in determining the parameters of the suspension filtration model based on multi-stage kinetics in porous media.

It should be noted, that parallel computing is a calculation that can be performed on multiprocessor systems using the ability to simultaneously perform many actions that are generated in the process of solving one or more tasks (one project). The main goal of parallel computing is to reduce the time to solve the problem. The task of parallel computing is to create a source of parallelism in the processes of solving problems in order to achieve the most efficient use of multiprocessor calculations (getting a parallel algorithm). Parallel work is work that allows you to do it at the same time (not necessarily independently). A parallel algorithm is an algorithm that can be executed simultaneously (not necessarily independently); an operation or set of operations to be performed at the same time must be expressly or impliedly indicated.

Referred to in paragraph above, we analyze in detail the developed numerical algorithm for finding coefficients D_L, β_a, β_d, β_p and find parts that can be parallelized from it.

1. This step is very simple, so there is no need to parallelize it.

2. At this stage, five differential equations are being solved, and what is important is that the solutions of these 5 systems are not related to each other, which means that they can be divided into at least 5 separate tasks and calculations can be carried out in parallel.

3. The system (18)-(20) consists of a system of 4 unknowns, 4 linear algebraic equations and it is solved by Cramer's method. To solve this system, 5 4x4 determinants are calculated, and their calculation is independent, so this step can also be divided into 5 subproblems. It should be noted that the calculation of determinants of size 4x4 requires a lot of resources, in fact, it is possible to calculate it in parallel. But we speed up the calculation of determinants based on Python's internal capabilities. We will give information about this below.

4. This step is also very simple, so there is no need to parallelize it.

5. This step is also very simple, so there is no need to parallelize it.

Now, to make this algorithm more understandable, we will describe it in the form of a block diagram.

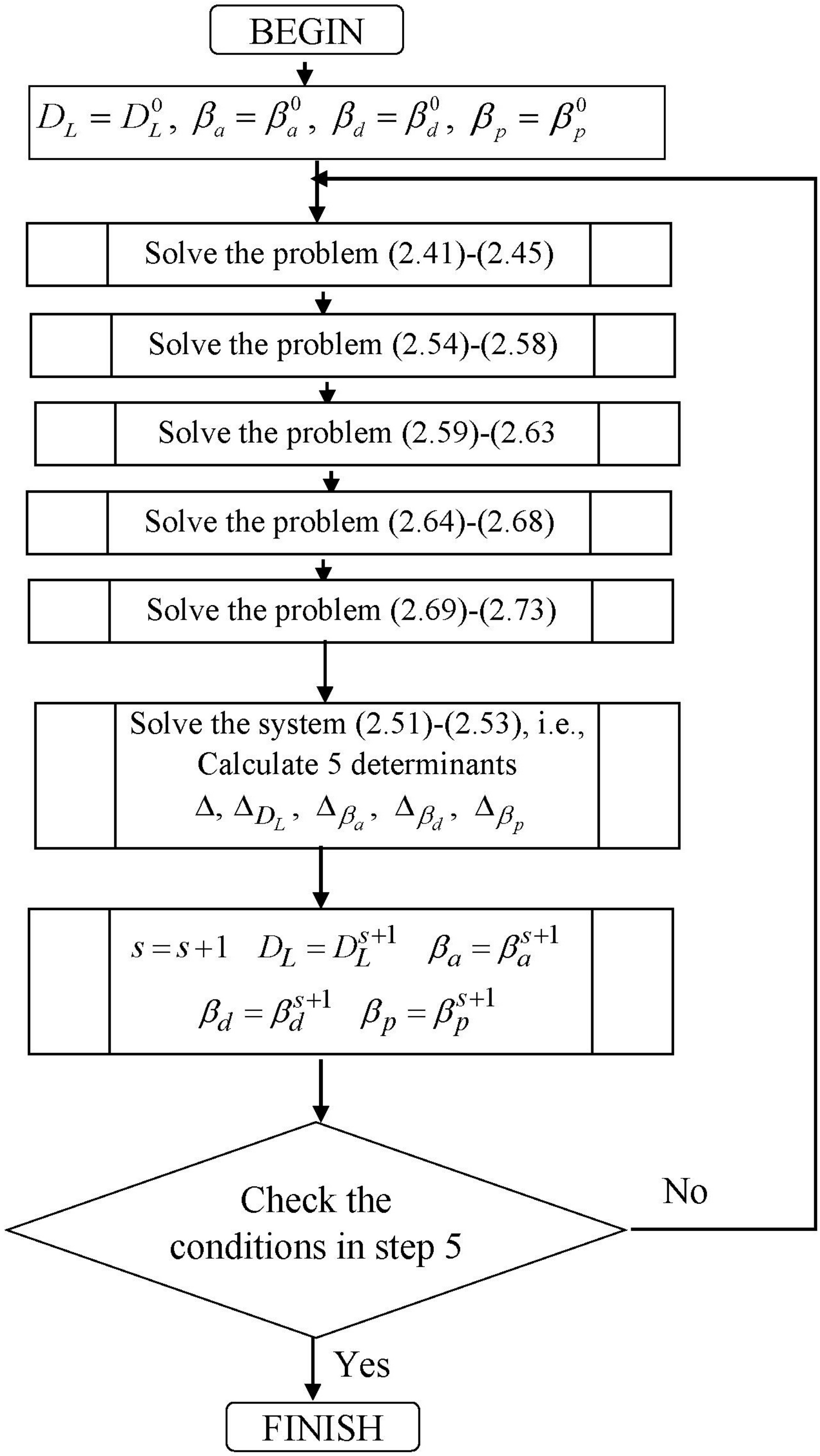

Figure 1 Algorithm for solving the problem without considering parallelization.

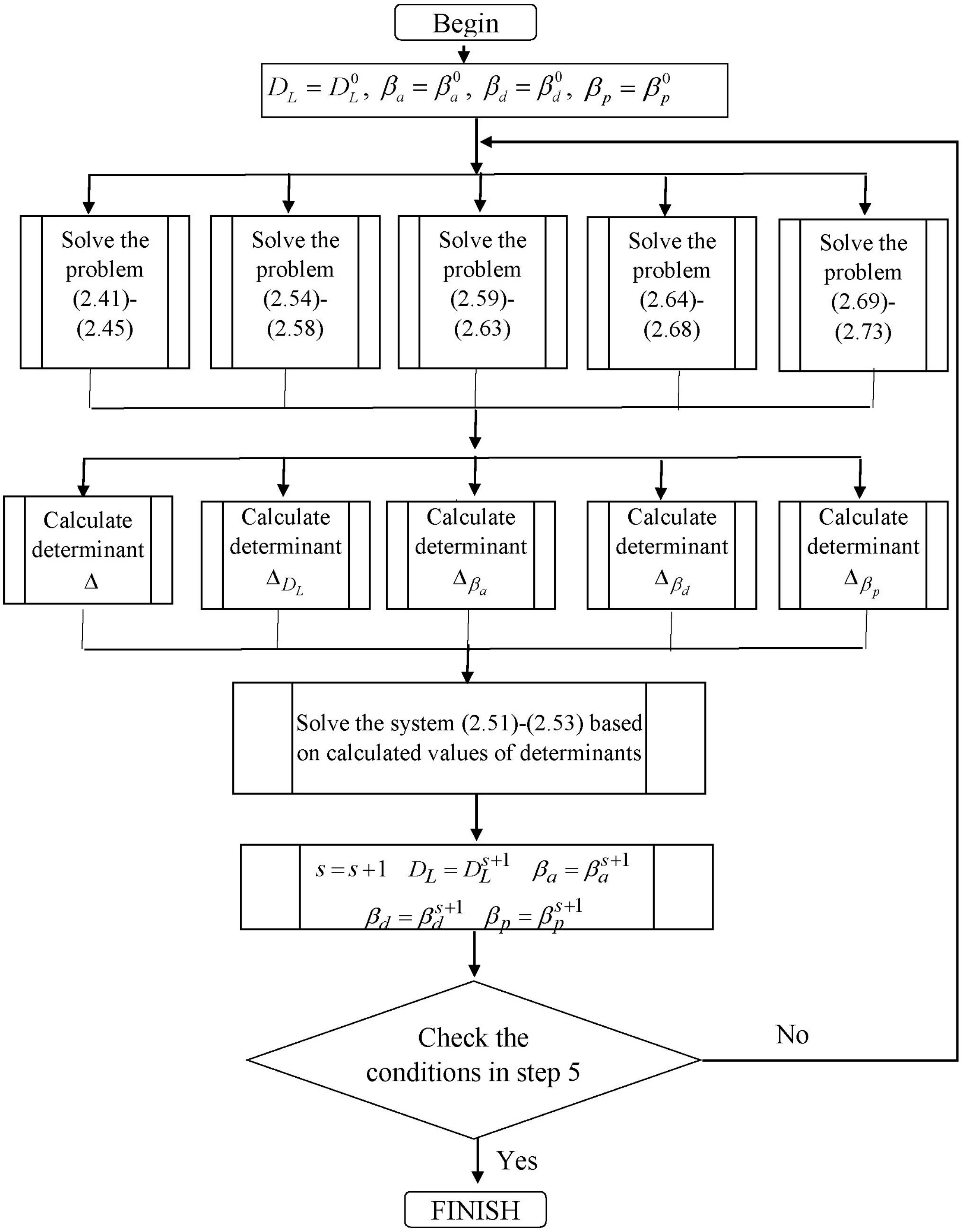

Figure 2 Algorithm for solving the problem with consideration of parallelization

4 Results and discussion

In order to solve the inverse problem and to carry out the numerical experiments a program developed in Matlab was created. As initial values, we take the following numeric values of the parameters [9]: $D_L = 0.033\,cm^2/s = 3.3 \cdot 10^{-6}\,m^2/s$, $L = 32\,cm = 0.32\,m$, $\beta_p = 5.19\,h^{-1} = \frac{5.19}{3600}\,s^{-1} \approx 1.4417 \cdot 10^{-3}\,s^{-1}$, $\beta_a = 9.48\,h^{-1} = \frac{9.48}{3600}\,s^{-1} \approx 2.6333 \cdot 10^{-3}\,s^{-1}$,

$\beta_d = 29.2\, h^{-1} = \frac{29.2}{3600}\, s^{-1} \approx 8.1111 \cdot 10^{-3}\, s^{-1}$, $c_0 = 6\, g/l = 6\, kg/m^3$, $d_m = 1.58\, \frac{g}{cm^3} = 1580\, \frac{kg}{m^3}$, $m = 0.393$.

First we solve problem (1)-(5) to prepare additional information (6) on the basis of quasi-real experiment for solving the inverse problem. Results of quasi-real experiment as values $z(t_j)$ given in Fig.3.

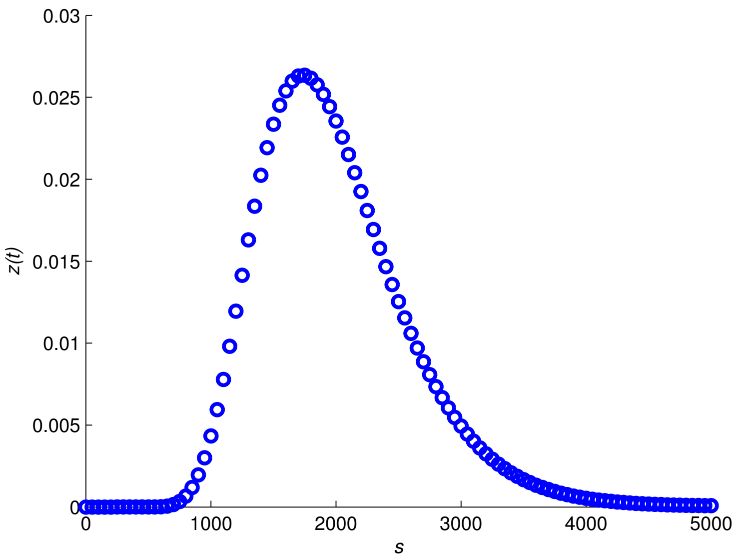

Figure 3 Function $z(t)$

Numerical calculations were carried out based on the above algorithm for solving the inverse problem. The results of calculations are shown in Figures 4-11.

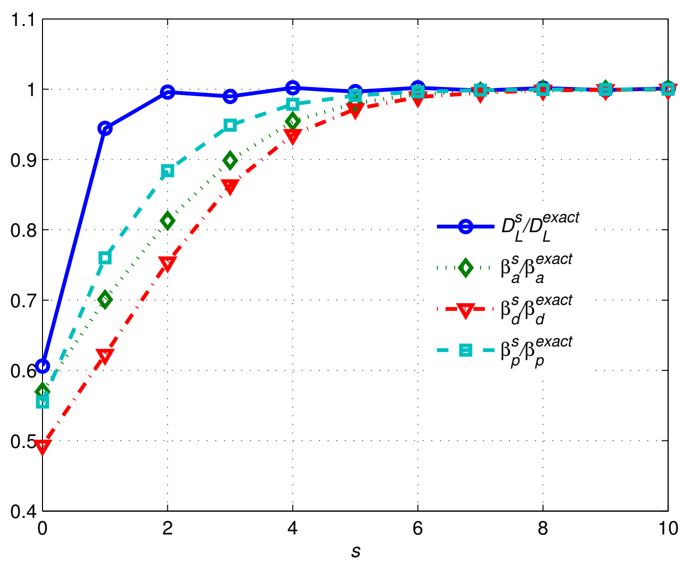

Figure 4 Recovery of coefficients D_L, β_a, β_d, β_p around the equilibrium point with initial approximations close to exact values of given parameters

Figure 4 shows how the coefficients recovered around the equilibrium point with initial values near to the equilibrium point. Fig. 5 shows each coefficient recovery in a separate plot. The calculation results show that all parameters have been restored with sufficient accuracy. It took 6 up to 10 iterations to recover the coefficients with initial values near to the equilibrium point (Fig. 4, 5). Figure 6 shows how the coefficients recovered around the equilibrium point with initial values a little away from the equilibrium point. Figure 7 shows each coefficient recovery in a separate plot. The calculation results show that all parameters have been restored with sufficient accuracy. It took 8 up to 20 iterations to recover the coefficients with initial values slightly distant to the equilibrium point (Fig. 6, 7).

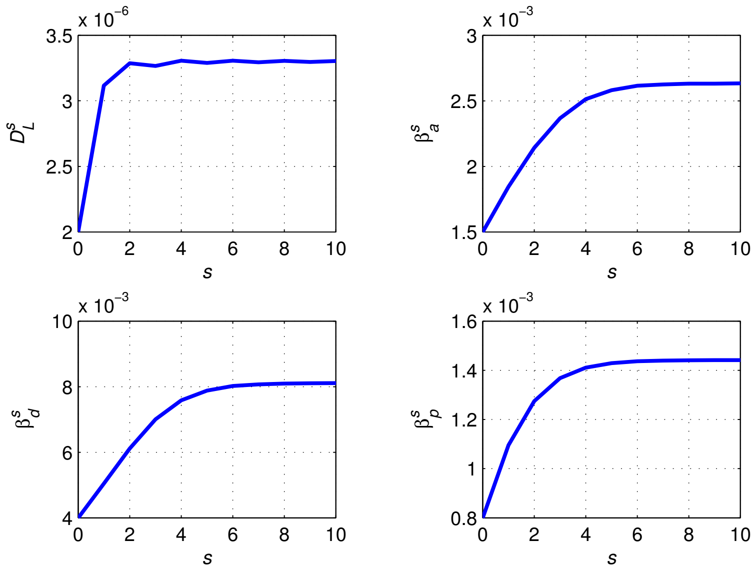

Figure 5 Recovery of coefficients D_L, β_a, β_d, β_p with initial approximations close to exact values of given parameters

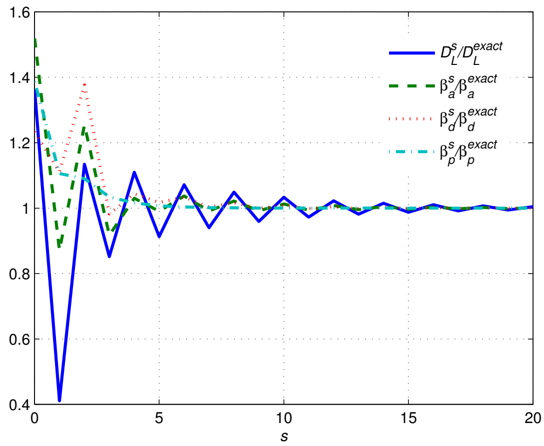

Figure 6 Recovery of coefficients D_L, β_a, β_d, β_p around the equilibrium point with initial approximations slightly distant from exact values of given parameters

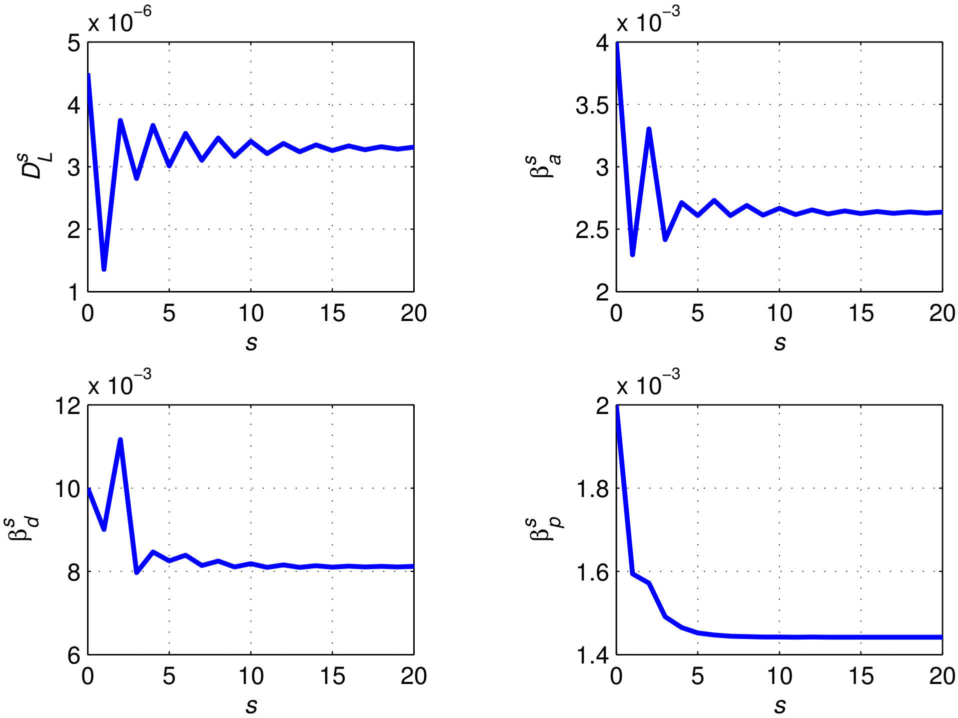

Figure 7 Recovery of coefficients D_L, β_a, β_d, β_p with initial approximations slightly distant from exact values of given parameters

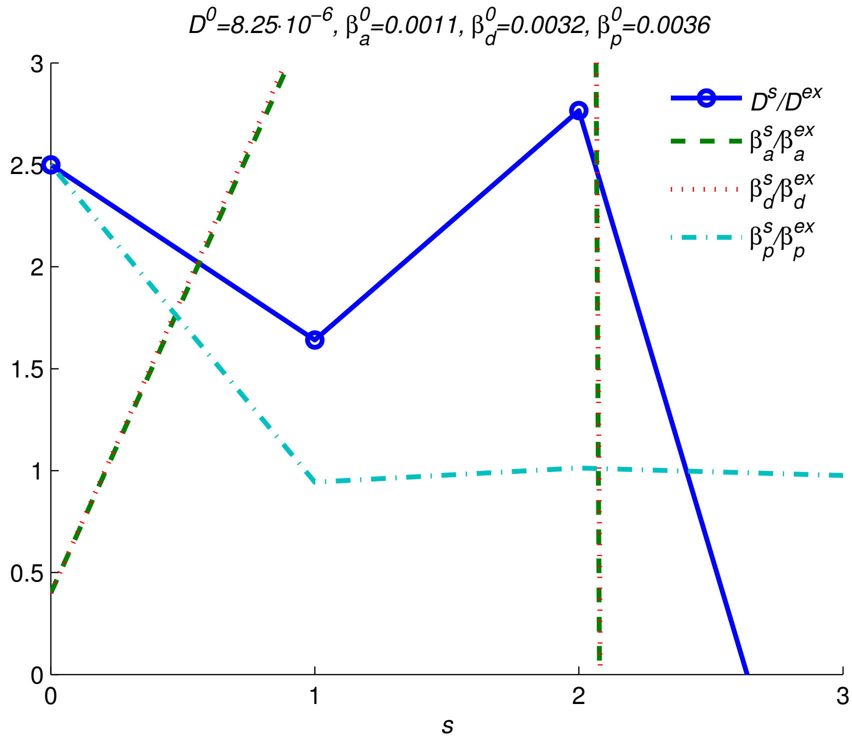

Figure 8 Recovery of coefficients D_L, β_a, β_d, β_p with initial approximations sufficiently far from the exact values of given parameters

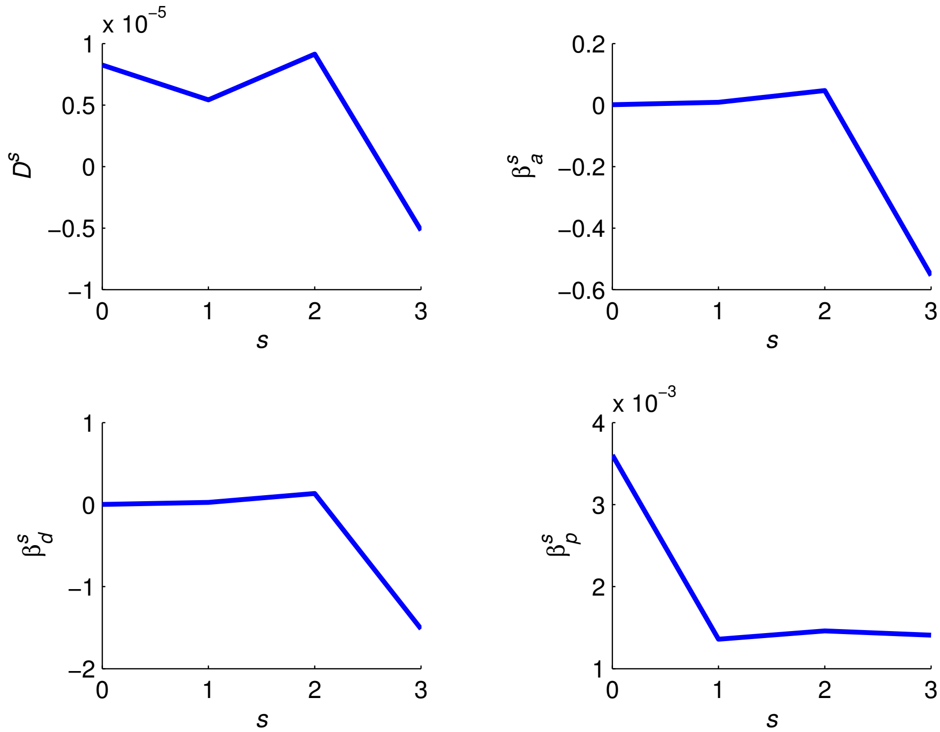

Figure 9 Recovery of coefficients D_L, β_a, β_d, β_p with initial approximations sufficiently far from the exact values of given parameters

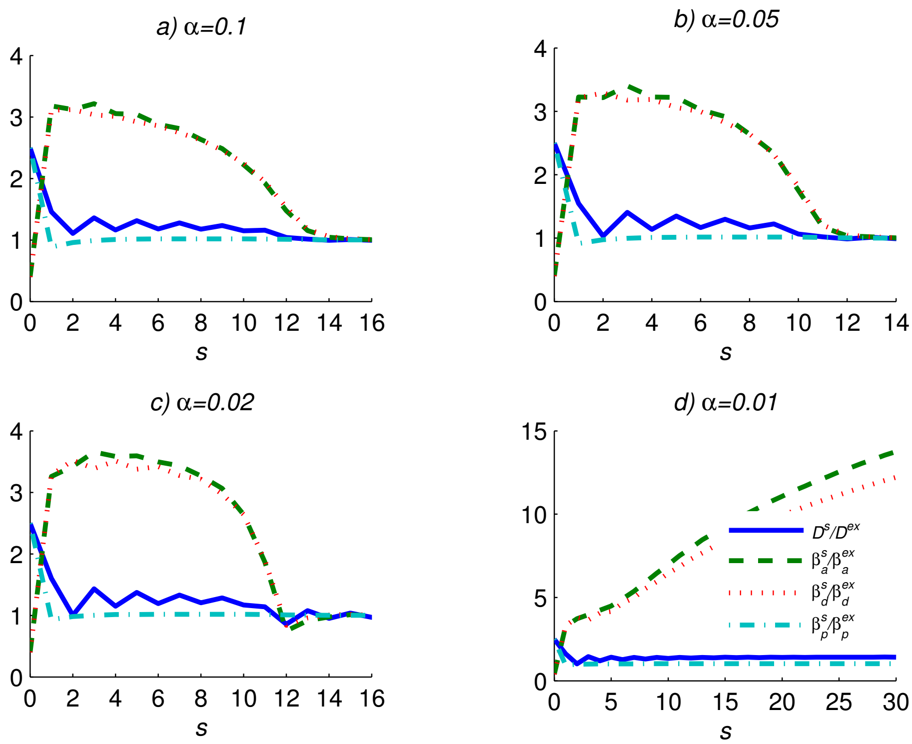

Figure 10 Recovery of coefficients D_L, β_a, β_d, β_p with different values of α

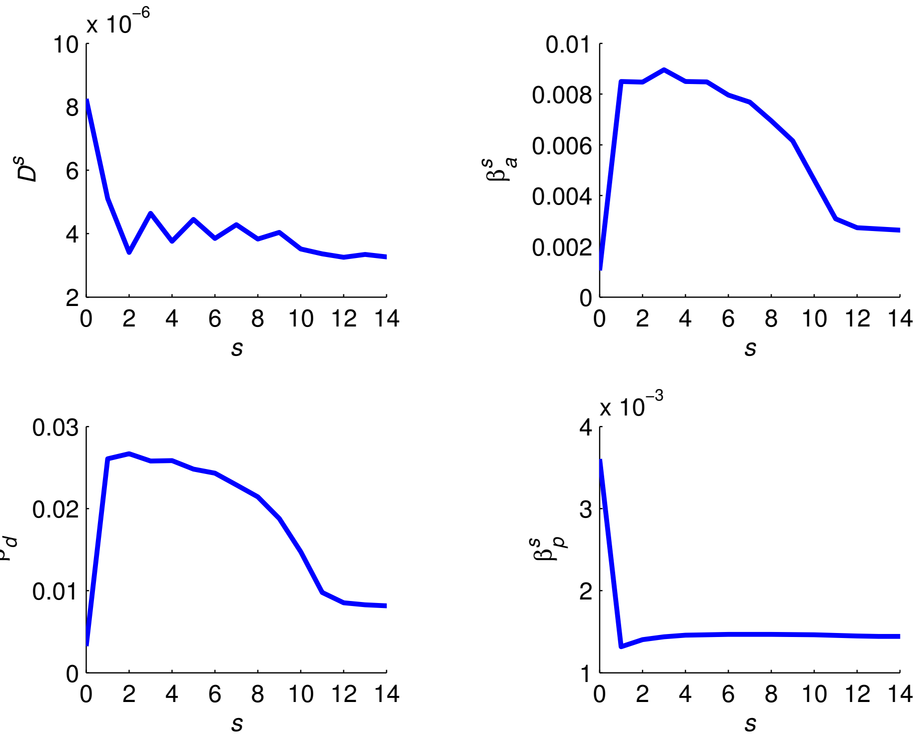

Figure 11 Recovery of coefficients D_L, β_a, β_d, β_p at optimal values of α

Figure 8 presents the results of calculating parameters at remote initial approximations from the equilibrium point. In this parameters restored with a sufficient accuracy. The calculation results show that when the initial approximations of the parameters are sufficiently far from the exact values of given parameters, the first-order identification method does not give good results, and the iterative process becomes divergent (Fig. 9).

So, we should improve the algorithm in order to get sufficient results. For a satisfactorily recovery of parameters at remote initial approximations we use the modified first order method [18]. At each iteration layer, instead of the functional (14) we use is made of the following functional:

$$\Phi\left(\gamma^{s+1}\right) = \Phi\left(\gamma^{s}\right) + \alpha\left(\gamma^{s+1} - \gamma^{s}\right)^{2}. \tag{41}$$

Figure 10 shows the results of identification parameters with remote initial approximations from the equilibrium point at various values of α. As can be seen from the figure, with a remote initial approximation of parameters from the equilibrium point at different values of the parameter α, parameters are restored with a sufficient accuracy. The calculation results show (Fig. 10) that as the initial approximation moves away from the equilibrium point, the required number of iterations increases. In this series of calculations, the value of the parameter $\alpha = 0.05$,is optimal. Therefore, in calculations with remote initial data, this value of the parameter was used (Fig. 11).

An experiment on computer also, carried out with developed parallel algorithm. We performed calculations only on four and eight core laptops. Using this algorithm allowed us to save time by 1.7 times when using 4-core laptops, and 2.6 times when using 8-core computers. It should be mentioned that in the system of differential equations (1)-(3) the (2) and (3) differential equations can be solved by means of parallelization, because they are not directly related to each other, that is, they are related to each other through the

(1) equation of the system, so parallelization can be used to solve them as well. Similarly, there are certain parallelization methods that can be used for solving the finite difference method, but since We performed calculations only on four and eight core laptops we were limited to the above parallelization algorithm in solving this problem.

5 Conclusion

A problem of identifying the parameters of a suspension filtration model within a porous medium is addressed through an inverse problem, which is successfully solved using numerical methods. Supplementary data for resolving the inverse problem is acquired through a quasi-real experiment involving the solution of the direct problem via the finite differences method. The resolution of the problem is achieved using the firs-order identification method. Notably, it is observed that deviating the initial approximations of the desired parameters from the equilibrium point leads to an increase in the number of iterations. The iteration count ranges from six to twenty, contingent on the specific choices of initial approximations. The study revealed that when the initial approximations of parameters deviate significantly from their exact values, the first-order identification method gives unsatisfactory results, leading to a divergent iterative process. In such instances, a modified identification method incorporating regularization was employed to successfully recover the parameters with sufficient accuracy. A parallel algorithm for the learned problem also developed. It allowed to decrease the calculating time up to 2.6 times.

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ПАРАЛЛЕЛЬНЫЙ АЛГОРИТМ ИДЕНТИФИКАЦИИ ПАРАМЕТРОВ МОДЕЛИ ФИЛЬТРАЦИИ СУСПЕНЗИИ В ПОРИСТОЙ СРЕДЕ

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В статье исследована математическая модель фильтрации суспензии в пористой среде, включающая уравнение баланса массы взвешенных частиц и кинетические уравнения как необратимого, так и обратимого осаждения частиц. Была сформулирована и решена численно обратная задача для определения сразу четырех параметров модели. Четыре параметра, которые необходимо найти: коэффициент диффузии в уравнении баланса массы, коэффициенты скорости осаждения в кинетических уравнениях как активных, так и пассивных зон и коэффициент обратного уноса обратимых осаждений. Для этой цели был использован метод идентификации первого порядка. Результаты показывают, что когда начальные приближения близки к точке равновесия, параметры восстанавливаются за небольшое количество итераций. При незначительном отклонении начальных приближений от точки равновесия количество итераций, необходимых для восстановления параметров, увеличивается, но коэффициенты восстанавливаются с достаточно малой погрешностью. Установлено, что при достаточном удалении начальных приближений параметров от точки равновесия метод идентификации первого порядка не дает хороших результатов и итерационный процесс становится расходящимся. В данном случае для восстановления параметров использовался модифицированный метод идентификации с использованием регуляризации, и параметры были восстановлены с достаточной точностью. Учитывая, что при решении обратной задачи выполняется большой объем вычислений, был предложен параллельный алгоритм решения этой задачи. Было обнаружено, что программа, основанная на распараллеленном алгоритме, работает значительно быстрее исходной программы.

Ключевые слова: конечные разности, математическая модель, обратная задача, параллельный алгоритм, пористая среда, регуляризация, фильтрация.

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