UDC 519.6

NUMERICAL MODEL AND COMPUTATIONAL ALGORITHM FOR SOLVING THE PROBLEM OF FILTRATION OF UNCONFINED GROUNDWATER

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This article presents the results of studies on the theory of filtration in an accurate forecast of changes in the level of groundwater during irrigation in the republic, as well as in assessing the impact of artificial and natural drainage structures on changes in the level of groundwater and one of the most important aspects of complex research - mathematical modeling of water layers with the use of various mathematical models of the theory of filtration, as well as the effective use of digital and computer modeling in solving problems of the theory of filtration.

Keywords: groundwater level, water confinement, filtration coefficient, infiltration, filtration zone.

Citation: Nazirova E., Shukurova M. 2024. Numerical model and computational algorithm for solving the problem of filtration of unconfined groundwater. *Problems of Computational and Applied Mathematics.* 1(55): 98-110.

1 Introduction

In our republic, studies on the theory of filtration are of particular interest when correctly predicting changes in the level of groundwater during irrigation, as well as assessing the effect of artificial and natural drainage structures on fluctuations in the level of groundwater are important for hydrogeology, militia and soil science.

According to the results of numerous studies of hydrogeological sections of strata, it has been established that in most cases the main water-bearing horizon from which pumping is performed is overlain by a low-permeable cover stratum from above, and underlain by a low-permeable layer from below, through which there is a connection with the underlying aquifers [1].

Optimal control problems for linear systems with quadratic performance criteria involve the solution of linear two-point boundary-value problems. For nonlinear optimal control problems, iterative methods must be used to obtain the numerical solutions of the nonlinear two-point boundary-value problems. These methods can also be used for linear or nonlinear optimal control problems subject to integral constraints [2].

The problem of the macroscopic simulation of the motion of a viscous fluid and mass transport in a porous medium is considered under the assumption that the mass transport can locally be described by the Fick relaxation law. Several cases determined by the local inertia number of the mass flow and the Péclet number are investigated. The macroscopic transport models are analyzed and compared with well-known phenomenological models [4].

On the basis of the derivation of the fADE, a heat transfer model using fHTE is developed for the characterization of geothermal reservoirs. We find that the fracture density in a geothermal area can be described by a power-law approximation with distance from the fault core. The fADE introduces a fractal diffusion coefficient, which decreases with distance from a high permeability zone in a reservoir to result in the diffusion into the surrounding rocks [5].

The problem of hydrogeological calculations of aquifers, oil and gas fields has not yet been fully resolved. Mathematical difficulties encountered in this direction forced researchers to simplify and schematize the physical picture of water movement in reservoir conditions. However, the requirements for scientifically sound predictions of filtration theory are increasing.

The choice of optimal mathematical models of the process under study is unthinkable without a thorough analysis of quantitative estimates, various natural and artificial factors affecting the process under study. In this case, it is advisable to carry out such an analysis using a computational experiment. This process consists of the following stages: statement of the problem; mathematical model; computational algorithm; programming; analysis of the results obtained and verification of the adequacy of the mathematical model.

One of the most important aspects of integrated research is the mathematical modeling of aquifers using various mathematical models of the theory of filtration. A model of filtration of a two-component suspension in a porous medium with the formation of two types of sediment, differing in their structures and properties, has been constructed. On the basis of computational experiments for suspensions with contrasting particle fractions, the influence of the parameters of the flux densities of liquid and particles, which limit the mass transfer between various components of the suspension and sediments, on the filtration characteristics and properties of the resulting sediments is estimated [6].

The most effective operational means for solving such problems of filtration theory is numerical and computer modeling. This approach the “overlapping continua” or “double-porosity” model was utilized for modeling the interaction of solutes in pores and fractures in the fractured porous domain. Assuming that porous medium is fractal, the novel expression for a mass flux was derived. It was proved that the diffusive flux in such medium should be defined by the mixed fractional derivative with respect to emporal and spatial variables. This non-Fickian expression of the mass flux can model sub-diffusion in the complex porous medium of fractal geometry [8].

Transition in a porous and immobile liquid porous medium the adsorption of a substance leads to a delayed distribution of zones of concentration of internal mass transfer between moving liquid zones and between zones. Nonlinear adsorption at the same kinetic parameters leads to an increase in adsorption effects. Transient processes characteristic of adsorption, internal mass transfer and migration of matter lead to mutually complex transient processes. In particular, there is a steady state start delay for transients that need to be completed faster than others. In nonlinear kinetics, internal mass transfer is accelerated for the same analogous parameters [9].

Capillary model of a porous medium was used for construction of a closure relations that allows us to describe the formation of polymer-disperse components and their influence on the porous medium.The complexity of solving the problem of impact of polymer-dispersed systems on the process of displacement of oil by water from porous medium is related to the three-dimensionality, nonstationarity processes of filtration, as well as the need to calculate the fields of polymer concentration, pressure and saturation fields, structural changes in porosity and permeability at each time step and at each point in space.

All these factors require a significant computer time and resources. The paper presents a parallel implementation of the algorithm for multiprocessor computer system using the MPI library. Calculations on a real field have shown efficiency of the algorithm [11].

The equations for filtering suspensions describing the growth of a cake layer on the filter surface for the generalized nonequilibrium Darcy's law have been compiled. Hydrodynamic filtration problems for these equations are formulated and numerically solved. On the basis of numerical calculations, it is shown that with an increase in relaxation effects, all other things being equal, the growth of the sediment layer will become more intense. In a fixed place of the sediment, the compression pressure changes only slightly. The filtrate consumption also increases. It was also established that there is a significant compaction of the sediment near the border "sediment - filter". As a result, the permeability of the cake layer decreases, while the rate of this decrease increases with time [14].

In groundwater hydrology, geophysical imaging holds considerable promise for improving parameter estimation, due to the generally high resolution and spatial coverage of geophysical data. However, inversion of geophysical data alone cannot unveil the distribution of hydraulic conductivity. Jointly inverting geophysical and hydrological data allows benefitting from the advantages of geophysical imaging and, at the same time, recover the hydrological parameters of interest. We introduce a first-time application of a coupling strategy between geophysical and hydrological models that is based on structural similarity constraints. Model combinations, for which the spatial gradients of the inferred parameter fields are not aligned in parallel, are penalized in the inversion. This structural coupling does not require introducing a potentially weak, unknown and non-stationary petrophysical relation to link the models. The method is first tested on synthetic data sets and then applied to two combinations of geophysical/hydrological data sets from a saturated gravel aquifer in northern Switzerland. Crosshole ground-penetrating radar (GPR) travel times are jointly inverted with hydraulic tomography data, as well as with tracer mean arrival times, to retrieve the 2-D distribution of both GPR velocities and hydraulic conductivities. In the synthetic case, it is shown that incorporating the GPR data through a joint inversion framework can improve the resolution and localization properties of the estimated hydraulic conductivity field. For the field study, recovered hydraulic conductivities are in general agreement with flowmeter data [15–18].

The increasing demands on water resources in recent decades in semi-arid regions has motivated researchers to develop effective methods for water resources management to avoid shortages due to groundwater mining [22]. Further, the pressure of population growth, food industry, and energy production increases the challenges for decision makers to adopt robust methods for management that satisfy demand [23, 24].

Mirghani [25] used evolutionary strategies (ES) to identify the source of groundwater contaminants. The authors built a simulation-optimization approach to minimize the root square error between the observed and monitored concentration of pollution in certain observation wells. Ayvaz [26] implemented harmony search (HS) algorithm combined with a simulation model in groundwater management to optimize the pumping rates and costs. Safavi [27] coupled simulation and optimization models to minimize the deficit in irrigation water demands using an artificial neural network (ANN) and a genetic algorithm (GA).

Consideration of the unsteady movement of groundwater in an area with an inclined water-resistant, when free-flow filtration of groundwater with a free surface in a single-layer zone turns into pressure-free filtration in a layered zone. This takes into account the lateral inflows in the filtration area, and evaporation.

2 Mathematical model

Mathematically, such a problem reduces to integrating a quasilinear system of partial differential equations of parabolic type:

$$\left.\begin{aligned} &\frac{\mu_1}{k_1}\frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left[(H-b)\frac{\partial H}{\partial x}\right] + \frac{f_1}{k_1},\\ &\frac{\mu_2}{k_2}\frac{\partial Z}{\partial t} = \frac{\partial}{\partial x}\left[(Z-b)\frac{\partial Z}{\partial x}\right] + \frac{f_2}{k_2} - (1-\frac{h}{Z-b}),\\ &\frac{\mu_3}{T}\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial x^2} + \frac{k_2}{T}(1-\frac{h}{Z-b}) \end{aligned}\right\} \tag{1}$$

with appropriate boundary conditions:

$$H(x,0) = F_1(x),\ 0 \leqslant x \leqslant L_1; \tag{2}$$

$$\left.\begin{aligned} Z(x,0) = F_2(x),\\ h(x,0) = F_3(x), \end{aligned}\right\};\quad L_1 \leqslant x \leqslant L_2; \tag{3}$$

$$k_1(H-b)\left.\frac{\partial H}{\partial x}\right|_{x=0} = Q_1(t); \tag{4}$$

$$T\left.\frac{\partial h}{\partial x}\right|_{x=L_2} = Q_2(t);\quad \left.\frac{\partial Z}{\partial x}\right|_{x=L_2} = 0; \tag{5}$$

$$k_1(H-b)\frac{\partial H}{\partial x} = k_2(Z-b)\frac{\partial Z}{\partial x} + T\frac{\partial h}{\partial x};\quad x = L_1, \tag{6}$$

where:
$H(x,t),\ b(x),\ k_1,\ \mu_1,\ f_1$ – groundwater level, water confinement, filtration coefficients, free water loss and infiltration in a single-layer filtration zone, respectively;
$Z(x,t),\ b(x), k_2, \mu_2, f_2$ – the same for the cover stratum, water confinement, filtration coefficients, free fluid loss and infiltration into the stratified filtration zone, respectively;
$h(x,t), T, \mu_3$ – is the head, the coefficients of water conductivity and elastic water loss in the head horizon of the stratified filtration zone;
Q_1 and Q_2 – lateral tributaries to the filtration zone;
L_1 – zone boundary;
L_2 – length of the filtering area;
$F_1(x), F_2(x), F_3(x)$ are given functions.

By passing to dimensionless variables in system (1) under boundary conditions (2) - (6) by the formulas:

$$H = H * H_0;\ Z = Z * H_0;\ h = h * H_0;\ b = b * H_0;\ x = x * L_2;\ \tau = \frac{Tt}{\mu_3 {L^2}_2}.$$

Here H_0 is some characteristic level.

Omitting, for convenience, the asterisks for dimensionless variables, we obtain a dimensionless system of differential equations:

$$\begin{cases} \dfrac{\mu_1 T}{k_1\mu_0 H_0}\dfrac{\partial H}{\partial \tau} = \dfrac{\partial}{\partial x}(H\dfrac{\partial H}{\partial x}) - \dfrac{\partial}{\partial x}(b\dfrac{\partial H}{\partial x}) + f_1\dfrac{l_2^2}{k_1 {H_0}^2}, \\ \dfrac{\mu_2 T}{k_2\mu_0 H_0}\dfrac{\partial Z}{\partial \tau} = \dfrac{\partial}{\partial x}(Z\dfrac{\partial Z}{\partial x}) - \dfrac{\partial}{\partial x}(b\dfrac{\partial Z}{\partial x}) + f_2\dfrac{l_2^2}{k_2 H_0} - \dfrac{l_2^2}{{H_0}^2}(1-\dfrac{h}{Z-b}), \\ \dfrac{\partial h}{\partial \tau} = \dfrac{\partial^2 h}{\partial x^2} + \dfrac{k_2 l_2^2}{T H_0}(1-\dfrac{h}{Z-b}) \end{cases} \tag{7}$$

with boundary conditions:

$$H(x,0) = \varphi_1(x), \;\; 0 \leqslant x \leqslant l_1; \tag{8}$$

$$\left.\begin{array}{r} Z(x,0) = \varphi_2(x), \\ h(x,0) = \varphi_3(x), \end{array}\right\}; \;\; l_1 \leqslant x \leqslant l_2; \tag{9}$$

$$(H-b)\left.\frac{\partial H}{\partial x}\right|_{x=0} = Q_1^*; \tag{10}$$

$$\left.\begin{array}{r} \left.\dfrac{\partial h}{\partial x}\right|_{x=l_2} = Q_2^*; \\ \left.\dfrac{\partial Z}{\partial x}\right|_{x=l_2} = 0; \end{array}\right\} \tag{11}$$

$$T_1(H-b)\frac{\partial H}{\partial x} = T_2(Z-b)\frac{\partial Z}{\partial x} + \frac{\partial h}{\partial x}, \;\; x = l_1; \tag{12}$$

where

$$T_1 = \frac{k_1 H_0}{T}; \;\; T_2 = \frac{k_2 H_0}{T}.$$

3 Numerical algorithm

Construct in area $(0, l_2)$ a uniform mesh with a step Δx

$$\Omega_{\triangle x} = \{(x_i = i\cdot\Delta x), \;\; i = 1, N\}. \tag{13}$$

Approximating the differential operators in (7) - (12) by finite-difference analogs on the grid (13) and applying the quasilinearization method [2] to represent nonlinear terms, obtaining this:

$$(\tilde{H}_{i-1} - b_{i-0.5})H_{i-1}(R_1 G + 2H_1 - b_{i-0.5} - b_{i+0.5})H_i +$$
$$+(\tilde{H}_{i-1} - b_{i+0.5})H_{i+1} + (R_1 G\breve{H}_i -$$
$$-\frac{\tilde{H}_{i-1}}{2} + \tilde{H}_i^2 - \frac{\tilde{H}_{i+1}^2}{2} + R_2) = 0,$$

$$(\tilde{Z}_{i-1} - b_{i-0.5})Z_{i-1}(R_3 G + 2\tilde{Z}_1 - b_{i-0.5} - b_{i+0.5} + R_5\frac{\tilde{h}_i}{\tilde{Z}_i})Z_i +$$
$$+(\tilde{Z}_{i+1} - b_{i+0.5})Z_{i+1} + \frac{R_5}{\tilde{Z}_i}h_i +$$
$$+R_3 G\breve{Z}_i - \frac{\tilde{Z}_{i+1}^2}{2} + \tilde{Z}_i^2 + \frac{\tilde{Z}_{i+1}^2}{2} + R_4 - R_5(1 - \frac{h_i}{Z_i - b_i}) = 0,$$

$$h_{i-1} - (G + 2 + \frac{R_6}{Z_i - b_i})h_i - h_{i+1} + R_6\frac{h_i}{(\tilde{h}_i - b_i)^2}Z_i = -(\tilde{h}_i G + R_6(1 - \frac{\tilde{h}_i\tilde{Z}_i}{(\tilde{Z}_i - b_i)^2})),$$

where

$$R_1 = \frac{\mu_1 T}{k_1 \mu_3 H_0};\ R_2 = \frac{f_1 \triangle x^2}{k_2 H_0^2};\ R_2 = \frac{\mu_2 T}{k_3 \mu_3 H_0};\ R_4 = \frac{f_2 \triangle x}{k_2 H_0^2};$$
$$R_5 = \frac{\triangle x^2}{H_0^2};\ R_6 = \frac{R_2 \triangle x^2}{T H_0};\ G = \frac{\Delta x^2}{\Delta \tau}.$$

Note that the first equation of system (7) has the form

$$a_i H_{i-1} - b_i H_i + c_i H_{i+1} + d_i = 0, \tag{14}$$

where

$$a_i = \tilde{H}_{i-1} - b_{i-0.5};\quad c_i = \tilde{H}_i - b_{i+0.5};$$
$$b_i = R_1 G + 2\tilde{H}_i - b_{i-0.5} - b_{i+0.5};$$
$$d_i = R_1 G \tilde{H}_i - \frac{\tilde{H}^2{}_{i-1}}{2} + \tilde{H}^2{}_i - \frac{\tilde{H}^2{}_{i+1}}{2} + R_2;\ \ i = \overline{1, M}.$$

The second and third equations of this system are the system [2]:

$$\begin{cases} a_i' Z_{i-1} - b_i' Z_i + c_i' Z_{i+1} + d_i' h_i = -f_i'; \\ a_i'' h_{i-1} - b_i'' h_i + c_i'' h_{i+1} + d_i'' Z_i = -f_i'', \end{cases} \tag{15}$$

where

$$a_i' = \tilde{Z}_{i-1} - b_{i-0.5};\ \ b_i' = R_3 G + 2Z_i - b_{i-0.5} - b_{i+0.5} + R_5 \frac{\tilde{h}_i Z_i}{(\tilde{Z}_i - b_i)^2};$$
$$c_i' = \tilde{Z}_{i+1} - b_{i+0.5};\quad d_i' = \frac{R_5}{\tilde{Z}_i - b_i};$$
$$f_i' = R_3 G \breve{Z}_i - \frac{\tilde{Z}_{i+1}{}^2}{2} + \tilde{Z}^2{}_i - \frac{\tilde{Z}_{i+1}{}^2}{2} + R_4 + R_5 (1 - \frac{\tilde{h}_i \tilde{Z}_i}{(\tilde{Z}_i - b_i)^2});$$
$$a_i{}'' = 1;\ \ b_i'' = G + 2 + \frac{R_6}{(\tilde{Z}_i - b_i)^2};\quad c_i'' = 1;$$
$$d_i'' = R_6 \frac{\tilde{h}_i}{(\tilde{Z}_i - b_i)^2};\ \ f_i'' = \breve{h}_i G + R_6 (1 - \frac{\tilde{h}_i \tilde{Z}_i}{(\tilde{Z}_i - b_i)^2}).$$

We will seek a solution (14) - (15) in the form:

$$\begin{cases} H_i = A_i H_{i+1} + B_i; \\ Z_i = A_i' Z_{i+1} + B_i' h_{i-1} + C_i'; \\ h_i = A_i'' h_{i+1} + B_i'' Z_{i-1} + C_i'', \end{cases} \tag{16}$$

where

$$A_i = \frac{a_i}{b_i - c_i A_{i-1}};\quad B_i = \frac{d_i + c_i B_{i-1}}{b_i - c_i A_{i-1}}; \tag{17}$$

$$\begin{gathered} A_i' = \frac{c_i' (b_i'' - a_i'' A_{i+1}'')}{W};\ B_i' = \frac{c_i'' (a_i' B_{i+1}' + d_i')}{W}; \\ C_i' = \frac{(b_i'' - a_i'' A_{i+1}'')(a_i' C_i' + f_i') + (a_i' B_{i+1}' + d_i')(a_i'' C_i'' + f_i'')}{W}; \\ A_i'' = \frac{c_i'' (b_i' - a_i' A_{i+1}')}{W};\ B_i'' = \frac{c_i' (a_i'' B_{i+1}'' + d_i'')}{W}; \\ C_i'' = \frac{(b_i' - a_i' A_{i+1}')(a_i'' C_i'' + f_i'') + (a_i'' B_{i+1}'' + d_i'')(a_i' C_i' + f_i')}{W}; \end{gathered} \tag{18}$$

$$W = (b'_i - a'_i A'_{i+1})(b''_i - a''_i A''_{i+1}) + (a''_i B''_{i+1} + d''_i)(a'_i B'_{i+1} + d'_i).$$

In this case, A_0 and B_0 are found from the boundary condition (10).

Solving together the finite-difference analogue of condition (10) with equation (14) for $i = 1$, we have:

$$\begin{aligned} A_0 &= \frac{b(b_1 - 4c_1) + 4\tilde{H}_1 c_1 - \tilde{H}_2 b_1}{b(a_1 - 3c_1) + 3\tilde{H}_0 c_1 - \tilde{H}_2 a_1}; \\ B_0 &= \frac{0.5c_1(3\tilde{H}^2{}_0 - 4\tilde{H}^2{}_1 + \tilde{H}^2{}_2 + Q) + d_1(\tilde{H}_2 + b_1)}{b(a_1 - 3c_1) + 3\tilde{H}_0 c_1 - \tilde{H}_2 a_1}. \end{aligned} \tag{19}$$

Similarly, using boundary conditions (11), we obtain:

$$\begin{aligned} A'_N &= \frac{4C'_{N-1} - b'_{N-1}}{3C'_{N-1} - a'_{N-1}};\ \ B'_N = \frac{d'_{N-1}}{3C'_{N-1} - a'_{N-1}};\ \ C'_N = \frac{f'_{N-1}}{3C'_{N-1} - a'_{N-1}}; \\ A''_N &= \frac{4C''_{N-1} - b''_{N-1}}{3C''_{N-1} - a''_{N-1}};\ \ B''_N = \frac{d''_{N-1}}{3C''_{N-1} - a''_{N-1}};\ \ C''_N = \frac{f''_{N-1}}{3C''_{N-1} - a''_{N-1}}. \end{aligned} \tag{20}$$

Finally, approximating expression (12) on grid (13) and applying the method of quasi-linearization, we arrive at the following:

$$\begin{gathered} H_M = Z_M = h_M = \\ = \frac{W_1 B_{M-1} - W_2 W_{11} + W_7 - W_8 - W_3 C'_{M+1} + W_4 W_{12} - 8C''_{M+1} + 2W_{13}}{W}, \end{gathered} \tag{21}$$

where

$$\begin{gathered} W_1 = 8T_1(\tilde{H}_{M-1} - b);\ W_2 = 2T_1(\tilde{H}_{M-1} - b);\ W_3 = 8T_2(Z_{M+1} - b); \\ W_4 = 2T_2(\tilde{Z}_{M+2} - b);\ W_5 = 6T_1(\tilde{H}_M - b);\quad W_6 = 6T_2(Z_M - b); \\ W_7 = T_1(3\tilde{H}^2{}_M - 4\tilde{H}^2{}_{M-1} + \tilde{H}^2{}_{M-2});\ W_8 = T_2(3\tilde{Z}^2{}_M - 4\tilde{Z}^2{}_{M+1} + \tilde{Z}^2{}_{M+2}); \\ W_9 = A'_{M+1} + B'_{M+1};\ W_{10} = A''_{M+1} + B''_{M+1};\ W_{11} = A_{M-2}B_{M-1} + B_{M-2}; \\ W_{12} = A'_{M+2}C'_{M+2} + B'_{M+2}C''_{M+1} + C'_{M+2};\quad W_{13} = A''_{M+2}C''_{M+2} + B''_{M+2}C'_{M+1} + C''_{M+2}; \\ W = W_5 - A_{M-1}(W_2 A_{M-2} - W_1) - W_6 + W_9(W_3 + B''_{M+2}) - W_4(W_9 A'_{M+2} + \\ + W_{10}B'_{M+2}) - 6 - 2W_{10}(4 - A''_{M+2}). \end{gathered}$$

Successive approximations are found as follows.

Knowing from the boundary conditions $A_0, B_0, A'_N, B'_N, C'_N, A''_N, B''_N, C''_N$ by recurrent formulas (17) and (18), we obtain

$$A_i, B_i, (i = 1, 2, ..., M-1);\ A'_j, B'_j, C'_j,\quad A''_j, B''_j, C''_j (j = N-1, N-2, ..., M+1).$$

Then, calculating H_M, Z_M and h_M on (21) by formula (16), we obtain

$$H_j, Z_j, h_j (i = 1, 2, ..., M-1;\ j = N-1, N-2, ..., M+1).$$

The iterative process ends when the condition

$$\max\left\{\left|H_i{}^{(s)} - H_i{}^{(s+1)}\right|;\ \left|Z_i{}^{(s)} - Z_i{}^{(s+1)}\right|;\ \left|h_i{}^{(s)} - h_i{}^{(s+1)}\right|\right\} \leqslant \xi,$$

where ξ is a given small value, s is the number of iterations.

4 Computational experiment

For the developed algorithm, a program in the MathLab. C language has been compiled. With its help, the influence of the filtration parameters of aquifers on the dynamics of the distribution of levels and heads of groundwater in the zone of the boundary of the transition from a single-layer to a layered zone of the filtration area was studied.

In particular, an assessment was made of the influence of the filtration coefficients k_1 and k_2 on changes in levels and heads. The rest of the geofiltration parameters are considered constant.

The obtained forecast calculations for the second and fifth years after the start of irrigation are given in table 1.

Table 1. The obtained forecast calculations for the second and fifth years after the start of irrigation are given

№	**Filtration coefficient, m/day**		**Forecast levels at the border of the crossing (in meters)**	
	Single layer zone k_1	Layered zone k_2	for 2 years	for 5 years
1	5	0,5	431,958	441,026
2	2,5	0,5	427,506	433,689
3	0,5	0,5	421342	422,961
4	0,5	2,5	418,752	418,343

Table 2. Values of filtration coefficients of single-layer and layered zones

№	**Gravitational fluid loss.**		**Elastic layered fluid loss zone** μ^*	**Forecast levels at the border of the crossing (in meters)**	
	Single layer zone μ_1	Layered zone μ_2		for 2 years	for 5 years
1	0,15	0,10	0,08	422,342	423,961
2	0,15	0,10	0,003	419,803	422,417
3	0,15	0,15	0,15	421,318	422,920
4	0,10	0,35	0,08	423,698	425,218

From Table 2 it can be seen that the greater the difference in the values of the filtration coefficients of the single-layer and layered zones, the greater the rise in the groundwater level at the border of the transition (the initial level is 421.00).

Further, the influence of changes in the capacitive properties μ_1,μ_2 and μ^* on the change in the level of groundwater at the border of the transition is considered. The rest of the geofiltration parameters remain unchanged. The results of the numerical experiment are shown in table 2.

5 Conclusion

The analysis of the results obtained shows that the greater the difference in the values of the capacitive properties of the two zones, the greater the height of the forecast level at the border of the transition, while the accumulation of groundwater reserves in a single-layer free-flow zone will occur.

Currently, the authors are investigating even more complex mathematical models of aquifers, when two well-permeable heterogeneous aquifers, connected by a low-permeable layer, interact with soil flows in the cover strata through a cofferdam, taking into account infiltration, evaporation and the work of drainage structures.

Thus, a thorough analysis of various models of aquifers and theories of groundwater movement in them made it possible to effectively apply the developed algorithms and programs for calculating many real objects and to give practical recommendations for improving the reclamation state of old-irrigated and developed territories of the arid zone.

The algorithms described in previous chapters are implemented in the form of universal programs written in the algorithmic language Matlab.

With the help of these programs, various forecasting problems of pressure and non-pressure water are solved for a single-layer model of aquifers, taking into account all natural and artificial factors affecting the filtration process. The initial information for the programs is prepared as follows. The mesh filtering area is expanded to a rectangle with fictitious nodes. The input procedure must ensure that the values of levels (pressures) and filtration coefficients (water conductivity) are entered into two-dimensional arrays with the identifier HN, (TN) (the values of these functions in fictitious nodes are arbitrary).

All collected information about the initial conditions of the filtration area, i.e., about the position of the groundwater level, boundary conditions, design parameters and others, constitute an information array that is constantly increasing, expanding and supplemented with more detailed and accurate new data.

Consequently, along with the correct collection of information, it is necessary to provide the most rational forms of storage in types of technical media that are convenient for prompt entry into a computer.

The development of stable computational schemes, universal and efficient algorithms and software that allow solving problems of pressure and non-pressure filtration significantly increases the reliability of the resulting numerical calculations and their visualization in a graphical form of the object.

Therefore, in order to increase the efficiency of using modern computers, it is necessary to create software for conducting a computational experiment with interaction with computer specialists. This is necessary for making final or intermediate decisions in the process of analysis and monitoring.

Based on a mathematical model and calculation algorithm using the Matlab software tool, software has been developed to solve problems of groundwater filtration. The software interface for solving pressure filtration problems is shown in Fig.1.

For the computational experiment, the following initial data were used:

$n = 21$ - number of points in the grid area by x and y;

$nt = 500$ - total time for calculation;

$dt = 1$ - time step (per day);

$Pn1 = 200$ - initial pressure in the formation;

$k = 1$ - filtration coefficient;

μ - free water yield coefficient;

$Lx = 4000$ - length of the filtration area in the x and y directions;

$Q = 300$ - well flow rates;

$h = 10$ - power layer.

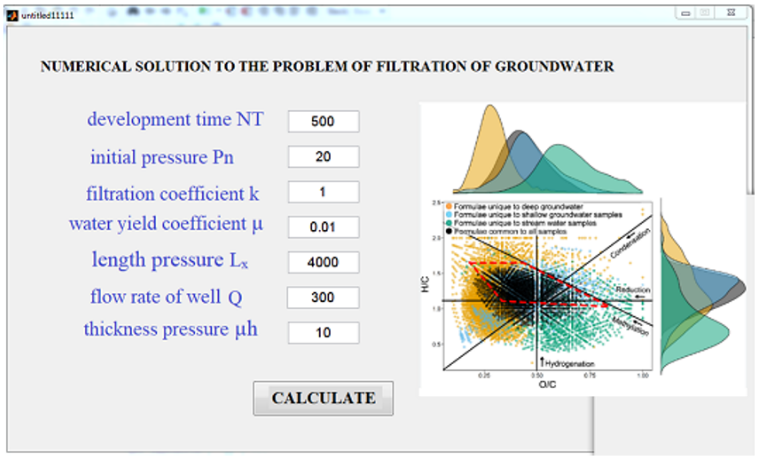

Figure 1 Program for solving problems of groundwater filtration and visualization of numerical calculations of a computational experiment

The calculation results for the problem of pressure filtration in a single-layer zone are displayed in the following form (Fig. 2):

- Change in pressure in the reservoir 3D graphics in various forms;
- Change in pressure in the reservoir in contour graphics;
- Change in pressure graphs in section along x;
- Pressure drop in well graphs.

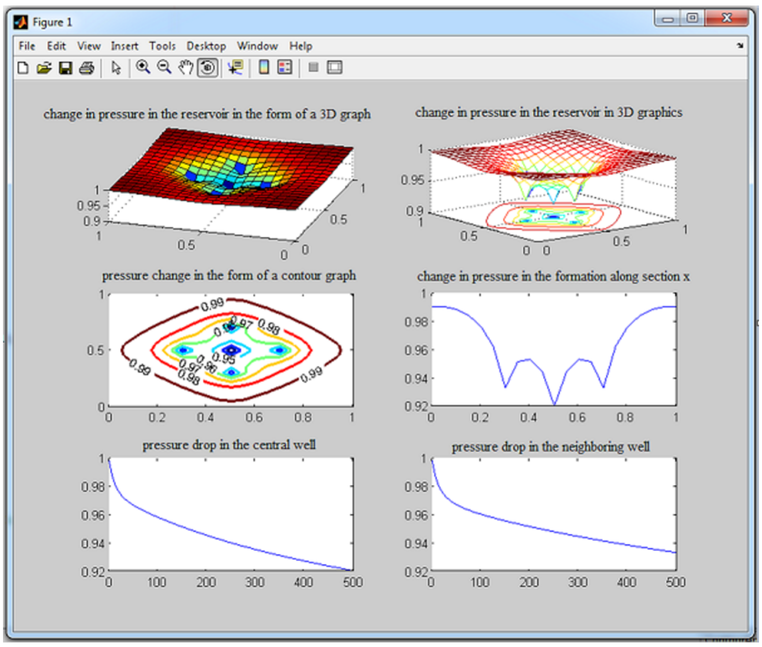

Figure 2 The results of the calculation of the computational experiment in visual form.

The software can be used for various similar two-dimensional problems, which mathematical model is described in the form of a differential equation of parabolic type.

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Received February 02, 2024

УДК 519.6

ЧИСЛЕННАЯ МОДЕЛЬ И ВЫЧИСЛИТЕЛЬНЫЙ АЛГОРИТМ РЕШЕНИЯ ЗАДАЧИ ФИЛЬТРАЦИИ БЕЗНАПОРНЫХ ГРУНТОВЫХ ВОД

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В данной статье представлены результаты исследований по теории фильтрации при точном прогнозе изменения уровня грунтовых вод при орошении в республике, а также при оценке влияния искусственных и естественных дренажных сооружений на изменение уровня грунтовых вод и один из важнейших аспектов комплексных исследований - математическое моделирование водных слоев с использованием различных математических моделей теории фильтрации, а также эффективное использование цифрового и компьютерного моделирования при решении задач теории фильтрации.

Ключевые слова: уровень грунтовых вод, водонасыщенность, коэффициент фильтрации, инфильтрация, зона фильтрации.

Цитирование: *Назирова Э., Шукурова М.* Численная модель и вычислительный алгоритм решения задачи фильтрации безнапорных грунтовых вод // Проблемы вычислительной и прикладной математики. – 2024. – № 1(55). – С. 98-110.

PROBLEMS OF COMPUTATIONAL AND APPLIED MATHEMATICS

No. 1(55) 2024

The journal was established in 2015.
6 issues are published per year.

Founder:
Digital Technologies and Artificial Intelligence Development Research Institute.

The journal is registered by Agency of Information and Mass Communications under the Administration of the President of the Republic of Uzbekistan.
The registration certificate No. 0856 of 5 August 2015.

ISSN 2181-8460, eISSN 2181-046X

Address:
100125, Tashkent, Buz-2, 17A.
Tel.: +(99871) 231-92-45.
E-mail: journals@airi.uz.
Web: www.pvpm.uz (journals.airi.uz).

Design and desktop publishing:
Sharipov Kh.D.

DTAIDRI printing office.
Signed for print 29.02.2024
Format 60x84 1/8. Order No. 1. Printed copies 100.

Contents